



JÖRGEN JERSILD

**ADVANCED
RHYTHMIC STUDIES**

POLYRHYTHMIC

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CHESTER MUSIC

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The reproduction on the cover is the so-called *Guidonian Hand*. It represents the oldest educational device in the science of music reading and is attributed to the Italian music theorist Guido d'Arezzo (c. 995-1050). The various steps of the tone ladder are placed on the fingertips and fingerjoints of the left hand, starting from the tip of the thumb with the lowest tone (G) and from here going across the palm up along the little finger and across the four fingertips, going down the index finger. The steps then continue as a spiral, ending up at the middle finger. By pointing to a position in the palm of his left hand, the teacher was able to indicate to his students which tone should be sung and at the same time to demonstrate the structure of the tonal system, measured by hexachords.

PREFACE

Polyrhythm is a useful subject of study. Besides preparing the practising musician for situations which he will meet sooner or later in the course of his work, the discipline also has other aspects: by developing the ability to combine contrasting rhythmic patterns and to interpret "foreign" rhythms (counter-rhythms) to a given basic pulse, skill is achieved in distinguishing between subtle rhythmic nuances, and a strengthening of the general feeling for tempo is attained. One often hears polyrhythmic passages which are inexactly executed. However, when such patterns are perceived and reproduced in an organic way, one can sense the fascinating tension caused by the polyrhythm and the effect is quite striking.

While it has been more or less a matter of course that a pianist, organist or player of other keyboard instruments must acquire the necessary skills in this field, it is far from being the case that this discipline forms an equal part of the training received by wind and string players or singers. Even though these performers are required to cope only with their own parts in a polyrhythmic structure, their task is no less demanding; here, too, is a question of perceiving the overall tension brought about by the contrasting rhythmic patterns.

As will readily be seen, the book consists of two principal sections, an introductory text in which the various types of polyrhythm are dealt with in turn, and two collections of exercises. The first of these consists of a series of exercises in which a melodic part is sung while a rhythmic part is performed as an accompaniment, either by clapping or by audibly beating time. These exercises can, therefore, be used, *regardless of which instrument is being studied*. In the second collection the different types of polyrhythm are applied to a series of two-part pieces. These exercises can be performed on the *piano* or other keyboard instruments, in which case both component parts of the polyrhythm are executed by the same player. They are, however, intended also as material *for polyrhythmic ensemble playing*, where two or more instruments participate. Singing of the parts is also advisable where range and character make this possible.

The difficulty of a given polyrhythm depends to a great extent on the context. In the initial exercises and in the 2-part pieces a contrasting rhythmic part is often combined with a regularly flowing part, thus greatly facilitating the execution of the polyrhythm.

As supplementary exercises for the final stage, use can be made of my volume "30 Polyrhythmic Etudes" (Edition Wilhelm Hansen, Copenhagen 1976), a collection of two- and three-part pieces which contains a more thorough treatment of the fundamental types.

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The polyrhythmic combinations dealt with in this book set out to cover all the most characteristic types and to include only forms which are comprehensible and feasible in performance. In order to obtain the concentration necessary, I have considered it most appropriate to compose all the exercises myself. Needless to say, this material may be supplemented with examples from the literature of the piano and organ, from chamber music, modern orchestral music, art-songs etc.


The majority of the exercises have been provided with metronome markings, which as a rule indicate the tempo at which the various polyrhythms are easiest to perform. However, when the particular polyrhythm has been learnt, it is also useful to perform the exercises at other tempi adjacent to the given marking. Where the use of the metronome is suggested, as is often the case, the clockwork metronome with its relatively weak pulse-beat is most convenient to work with; it has the additional advantage that it can also mark tempi which are not indicated on the normal metronome scale. This becomes especially useful in connection with the special method I have employed for the more complicated types, which involves calculating the metronome markings for the various cross-rhythms, comparing them and thus facilitating performance.

In conclusion: in the text asterisks have been put beside the comments to certain exercises; this indicates that the particular exercise is rather more difficult and may therefore be left until a later run-through. The exercises in the last chapter have been designated in the same way; the polyrhythms treated in this section are rather more specialized and are primarily relevant in connection with the performance of contemporary music.

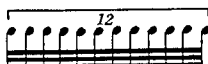
Jörgen Jersild

I. THE DIFFERENT TYPES OF SUBDIVISION AND THEIR NOTATION

If we imagine a given note-value divided into values of equal size — a duple, triple, quadruple, quintuple division etc. — then certain of these sub-divisions will represent *primary sub-divisions*, while the others, the *secondary sub-divisions*, are derivations produced by sub-dividing the primary types. The basic types, the primary sub-divisions, follow the prime series: 2, 3, 5, 7, 11 etc.; divisions into 4, 6 or 12 parts, for instance, are derived forms, as they can be traced back to the primary duple or triple divisions.

If we take a quintuple division, based on the quarter-note, the figure will, of course, look like this: ; the numbered bracket indicates the divisional type, and also signifies that the sixteenth-notes of the figure do not have their actual "face value". Note also the use of square brackets, which are to be preferred in order to avoid confusion with bowing or phrasing marks.

If we take as our next example a 12-fold division of the quarter-note, we shall also require a square bracket:



but it will be clear that the sub-division used here is one of the *secondary* kind, and it is impossible to discern directly where the accents fall in the figure. This method of notation is thus only appropriate in cases where the figure moves rapidly, the position of the accents thus being of little importance, or in cases where the accents clearly emerge from the context. If, however, it is necessary to show the accents, the notation must indicate the fundamental *primary* divisional type:



In the first case the principal division, as shown by the way the beams are joined, is a triple division; in the second case the figure divides into four sixteenth-notes which in turn are sub-divided into triplets, and the primary division is therefore duple. Note how in the first case only *one* tail is drawn right across, as this figure is based on the eighth-note triplet, while in the second case *two* tails are carried across, since in this case a sixteenth-note figure has been sub-divided.

Our system of rhythmical notation is constructed on the principle of duple division: ♩, ♪, ♫, ♬, ♭, ♮ etc. Here the individual notes stand for their actual face value. The other divisional types, triple, quintuple, septuple etc., must make do by borrowing from this binary series.

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Let us clarify this method of notation by sub-dividing the quarter-note up to a ten-fold division:

2-fold:			
3-fold:			
4-fold:			
5-fold:			
6-fold:		or:	
7-fold:			
8-fold:			
9-fold:		or:	
10-fold:		or:	


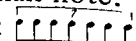
In these sub-divisions we employ three kinds of note-values: ♩, ♪ and ♫, corresponding within the binary series to a duplet, quadruple and octuple division respectively. The transference of these values to the other types of sub-division will inevitably cause inconsistencies. Note that the 9-fold division uses sixteenth-notes, i.e. a longer note-value than the 8-fold division, which is written with 32nd-notes, in spite of the fact that the 9-fold division represents the more rapid movement.



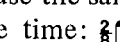

The triplet which uses the same note-value as the corresponding duplet would, when sub-divided further, rapidly “fall behind”; if each of the 16th-notes in the 9-fold division is sub-divided into 3, then this 27-fold division will be written in 32nd-notes, the same note-value which in the binary series is already reached at the 8-fold division.



The use of the same note-values in duplets and triplets can bring about other notational peculiarities. For example: if we wish to write down a triplet $\overbrace{\text{♩} \text{♩} \text{♩}}^3$, the last two notes of which are then replaced by another triplet, so that the half-note in this figure $\overbrace{\text{♩} \text{♩}}^2$ is divided into three, the resulting figure must be written thus: $\overbrace{\overbrace{\overbrace{\text{♩} \text{♩} \text{♩}}^3}^2}^3$; if the last two notes in this newly-formed triplet are replaced by yet another triplet – a rhythm which can certainly be performed – the figure will take on the following rather complex appearance: $\overbrace{\overbrace{\overbrace{\overbrace{\overbrace{\text{♩} \text{♩} \text{♩}}^3}^2}^3}^2}^3$.


We must still use the quarter-note value in each of the three triplets, in spite of the fact that they represent constantly increasing speeds. Having ascertained that figures which contain triple sub-divisions in their notation quickly “fall behind” in the system, we are in a position to establish the following basic rule for other divisional types – divisions into 5, 7, 11 parts etc. plus their sub-divisions: these types

employ the same note-value as the immediately preceding binary division. A quintuple division of the quarter-note uses sixteenth-notes, as does the quadruple division, and a septuple division also uses 16th-notes, because the quadruple division is the immediately preceding binary division in this case as well.

This rule can also be stated thus: when we transfer the note-values of the binary series to other divisional types, the constituent notes of the figure must add up to a greater "face value" than the value to which the figure as a whole corresponds. A half-note septuplet, for instance, is not written thus: , since the sum of the constituents notes' face value is only equal to 7 sixteenth-notes, i.e. one 16th-note too few in relation to the space which the figure occupies, i.e. a half-note. Therefore we must use the note-value one level above, i.e. the eighth-note: .

If we take a *compound* note value instead of the quarter-note — which is a *simple* note-value — as the starting-point for similar divisions, i.e. values such as $\overset{\frown}{\text{r}}$ or $\overset{\frown}{\text{p}}$, then the problems associated with the method of notation become somewhat complicated. Let us take as our first example the value $\overset{\frown}{\text{r}}$, which occupies a $\frac{3}{8}$ -measure, or half a $\frac{6}{8}$ -measure, and imagine a duple division of this note-value. According to one notational tradition, particularly characteristic of French music, the $\frac{3}{8}$ -duplet is written in eighth-notes: *. Similarly, the duplet division of the $\overset{\frown}{\text{r}}$ -value is written in quarter-notes: **. This method of notation follows the rule that duple and triple division use the same note-value; thus the first two bars in a passage like this, written in duple time:  || employ the same note-values when the figure is transferred to triple time:  ||.

A second method which is probably used more frequently is to write the $\frac{3}{8}$ -duplet in quarter-notes: |***, and the $\overset{\frown}{\text{r}}$ -duplet according to the same principle, in half-notes |****. This method complies with the rule that the sum of the face values of the constituent notes must not be less than the value on which the type of division is based.

A third method consists in the exact indication of the value of the notes making up the duplet, in $\frac{3}{8}$ thus |*****

These three methods of writing the duplet figure will, of course, influence the notation of the divisions into 4, 8, etc. In accordance with these three principles, the duplet, quadruplet and octuplet will be written thus:



The last of these possibilities (c) is attractive because of the precise indication it gives of the length of the notes. It is particularly suitable for use in cases where the

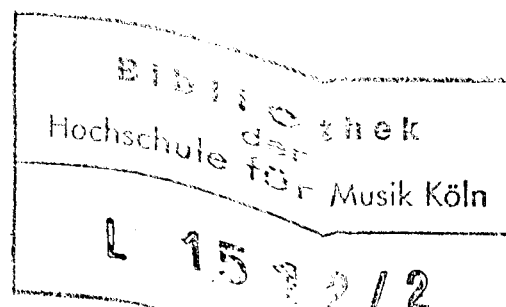
* See Fauré "A Clymène", Op. 58. 4 or "En bateau" from Debussy's "Petite suite".

** See Grieg's "To Spring".

*** See Hindemith's "Das Marienleben" no. 4.

**** See Brahms' Piano Sonata op. 1 (the last bars of the Scherzo movement).

***** See Debussy: "Iberia". p. 22 of the score (published by Durand).



divisional figure contains only identical note-values, or in a quadruplet figure like this: ; but if the quadruplet is further complicated, for instance by dotting the first of its four notes, we might tend to think that a notation like the following could be used: ; but it will be obvious that this is in fact quite wrong, as the *second* dot after the first note only increases its length by 1/4 of the *value of the sixteenth-note*, but not by half the *dotted* sixteenth-note, as is required in the present case.* Therefore this figure must be written thus: ; which demands a considerable degree of skill in handling fractions and the notation appears distinctly complicated.

Of the two methods of notation marked (a) and (b), the second must be regarded as the most suitable. If we choose the first, (a), the note-values used in the various divisions will be the same, regardless of whether \downarrow or \downarrow is the starting-point. Alternative (b), on the other hand, complies with the above-mentioned rule of surplus value and definitely seems to be preferable as the most logical and easy to read. On the basis of these considerations, therefore, the various sub-divisions of the value will be written as follows:

$\frac{3}{8} \downarrow$

2-fold:

3-fold:

4-fold:

5-fold:

6-fold: or: or:

7-fold:

8-fold:

9-fold:



10-fold: or: or:


It will be apparent that the note-values used here form a wholly satisfactory progressive sequence: the 9-fold division in fact fits into the sequence better than in the previous example, where the quarter-note was sub-divided. On the other hand, the sextuplet division shows a peculiarity, as the figure must naturally be written in sixteenth-notes if it contains *three* accents, while the figure with *two* accents


* This inaccurate figure can be found on page 418 of Joh. Wolf's "Handbuch der Notationskunde".

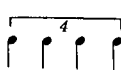
actually should be written in eighth-notes, as it represents a triple division of the duplet which has the value of a quarter-note. As the two figures move at exactly the same speed, it would be most practical, especially if the two figures occur side by side, to write the triplet division of the duplet in sixteenth-notes as well.


As a last example we shall take the sub-divisions of the $\frac{5}{8}$ -value, also in compliance with the rule of surplus value:




$\frac{5}{8}$ |  or: 

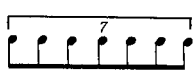
2-fold: 

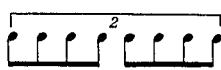
3-fold: 


4-fold: 

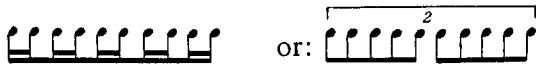
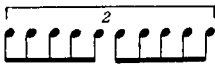
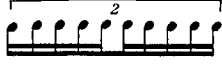
5-fold: 

6-fold:  or:  or: 

7-fold: 

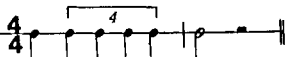
8-fold: 

9-fold: 

10-fold:  or:  or: 

The same phenomenon, as shown in the previous example, occurs here in the case of the sextuplet division, and furthermore in the case of the 10-fold division.

If we compare these three rows, the divisions of \downarrow , \downarrow and $\downarrow\downarrow$ we will find certain points of concurrence: e.g. in the first two rows the septuplet is written with the same note-value. However, such cases will not normally give rise to misunderstandings, as the time signature and the structure of the measure in question will show from which note-value the particular division is derived. Similarly is a case like this:

$\frac{4}{4}$ | 

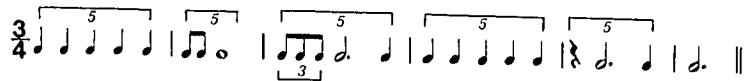
As the measure contains four quarter-notes, the quadruplet must be a sub-division of the \downarrow -value.

We can also employ this "subtraction method" in the less common cases in which a sub-divisional figure extends across a bar-line, e.g.:



The remainder in the first measure ($=\downarrow$) is added to the remainder in the second measure ($=\downarrow$); thus the quintuplet figure is a sub-division of the \downarrow -value. Note that the distribution of the single notes of the sub-division on either side of the bar-line is determined by when the note is struck — whether it comes *before* or *after* the bar-line break. If, in the present example, the figure is combined with another rhythm in quarter-notes for instance, it must be clear from the notation that the fifth note of the quintuplet does not coincide with the first beat of the second measure, but comes a little after it.

When the individual notes in a divisional figure enclosed in a numbered bracket are tied together to form larger units, or are further sub-divided, or when one or more of its values are replaced by rests, figures such as those shown in the next example can occur:



Figures of this kind are sometimes difficult to take in, but in respect of notation they do not pose any new problems. In this case the quintuplet division of the \downarrow -value is to be written as quarter-note quintuplets, which may thus form the same structures as the $\frac{5}{4}$ meter: a septuplet written in eighth-notes will thus produce the same structures as $\frac{7}{8}$ meter, and so on.

The principles demonstrated in these examples will be readily transferable to cases in which other note-values are sub-divided, and in which there are other divisional types than those dealt with here.

II. THE POLYRHYTHMS 3 AGAINST 2 AND 2 AGAINST 3

When rhythmic figures, each belonging to its own *primary* sub-divisional group, are simultaneously combined, the phenomenon of *polyrhythm* occurs.

A two-part musical structure is normally required for the formation of a polyrhythm; but a monodic melody line can also display a polyrhythmic relationship, i.e. in its relation to the basic rhythmic meter, as shown in the following passage:

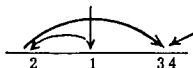


In quarter-note meter each of the two triplet figures in the second measure will form the polyrhythm 3 against 2 in relation to the basic meter.

This form, in which a recited or sung figure contrasts polyrhythmically with a given basic pulse beat, will be taken as the starting-point in learning the various polyrhythms.

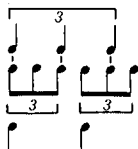
3 against 2

See Exercises p.33 no. 1. The relationship between the quarter-note triplets of the melody and the quarter-notes of the basic meter corresponds to the polyrhythm 3 against 2. In order to gain a clear impression of the polyrhythmic effect, this and subsequent exercises of the same kind will be accompanied by *beating time audibly*, i.e. the beats are simultaneously marked by tapping the table lightly: in this case, where, the time signature is $\frac{4}{4}$, thus:





Initially the metronome is also used as an aid. Furthermore, we will make it a rule to beat an introductory measure in all these exercises.

The timing of the *cross-rhythm*, the quarter-note triplet, on the basis of the meter of the introductory measure, may initially cause difficulties, but we can establish that both the contrasting rhythms “divide into” a triplet division of the quarter-note meter:

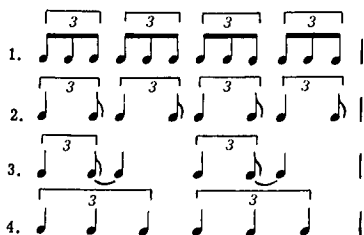


We can thus perform the first two notes of the quarter-note triplet by transcribing them mentally to this figure: $\overset{3}{\text{J}} \text{J} \text{J}$. If we perform the tied eighth-note perfectly rhythmically, so that it comes as a “gentle” *triplet* upbeat to the second beat (to be clearly distinguished from the corresponding binary figure: $\overset{3}{\text{J}} \text{J} \text{J}$), we will already be

able to perceive the polyrhythmic effect; and when the triplet figure has been commenced in this way, the third quarter in the triplet unit may be added without any difficulty.

When performing a polyrhythmic 3 against 2 figure it will become apparent that the triplet may tend to slow down the tempo. It is generally at the transition between the second and third notes of the triplet that the delay occurs. If the triplets are phrased thus: , the chances of delaying the third note will be increased, and it is therefore a help to start by imaging this phrasing: 

EXERCISE 1, p. 33. As a preliminary exercise to exercise 1, the metronome is set at $J=92$ and its pulse is followed by beating time audibly. When the two pulses (metronome and tapping) coincide perfectly, the series below is recited, repeating the fourth measure several times.



The exercise is also performed in more rapid tempi up to approx. 120, and then in slower tempi down to approx. 72. Finally measures 1-3 are omitted, and we work out the speed of the quarter-note triplets directly after the metronome has marked four beats.

Exercise 1 is then performed at tempo 92, accompanied by beating time audibly and initially with the use of the metronome as well; the auxiliary rhythms in measures 1-4 can, if desired, be included immediately beforehand.

EXERCISE 2, p. 33 is also performed with the metronome and beating time audibly. The exercises are also transferred to the instrument while the basic pulse is marked by the metronome: the pianist performs the melodic line doubled at the octave, strings play *if pizzicato*, winds with tonguing on each note. Choose a comfortable tempo (not too fast).

EXERCISES 3-9, p. 33. The degree of difficulty of the polyrhythmic triplet depends largely on the surrounding rhythmic context: when preceded by binary rhythmic figures, as in exs. 5-8, the timing of the triplet is more difficult than in exs. 3 and 4. It is important to check that the basic rhythm is maintained when the polyrhythm occurs, and is continued after it is played. The exercises should also be worked through without the metronome, only beating time audibly, and also performed instrumentally in the same way as ex. 2. Note the eighth-note meter in exercise 8 and the half-note meter in exercise 9. The exercises should also be sung through without a break, tempo = 92.

EXERCISE 10-13, p. 34. The various *derivations* of the polyrhythmic triplet do not cause great difficulties; the figure $\overset{3}{\text{J}}\text{J}\text{J}$ (in quarter-note meter) can be transcribed mentally to $\text{J}\overset{3}{\text{J}}\text{J}$ to avoid confusion with the corresponding binary figure JJ ; $\overset{3}{\text{J}}\text{J}\text{J}$ corresponds to the auxiliary rhythm used previously, $\overset{3}{\text{J}}\text{J}\text{J}$, and similarly $\overset{3}{\text{J}}\text{J}\text{J}$ can be transcribed to $\overset{3}{\text{J}}\text{J}\text{J}$. Transcriptions of this kind are especially useful in ensuring a more precise performance, especially at slower tempi. Note the eighth-note meter in exercise 11; ex. 13 is accompanied by beating $\frac{3}{4}$ -time audibly.

EXERCISE 14, p. 34. Repeat the repetition section of the exercise several times; the exercise is begun at tempo $\text{J} = \text{approx. } 52$ and the tempo is gradually increased to $\text{J} = \text{approx. } 132$ and is then decreased back to the original tempo. The accompanying eighth-notes are tapped or clapped.

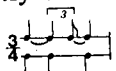
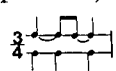
EXERCISES 15-18 p. 34: exercises with an accompanying part which is tapped or clapped. When singing the exercise through for the first time the melody part only is performed while beating time audibly in the basic meter (in ex. 15: quarter-notes, in 16: eighth-notes and so on). The exercise is then repeated, and the accompanying rhythmic part is tapped or clapped. The exercises are also performed so that each part is taken by a separate performer or shared among two groups, initially with the aid of the metronome. 3 against 2 polyrhythms occur here with both triplet and duplet in derived forms; in certain cases the derived types are combined, as in exercise 18 measure 3 and measure 7; when practising, the rest in the rhythm part in these measures may be replaced by the corresponding note.

EXERCISE 19, p. 36: 3 against 2 polyrhythm in $\frac{5}{8}$ time. The rhythmic accompaniment is beaten audibly (beats as in triple time, but the third beat short, corresponding to an eighth-note). The accompaniment part is first practised alone; to ensure that the eighth-notes at the end of each measure do not result in disruptions to the tempo (generally they will become too short). The metronome is used as a check; we use it to mark the quarter-notes at tempo 92, and the first two quarter-notes in all the odd measures (measures 1, 3, 5 etc.) will coincide with the metronome beat. The melodic part and the accompaniment part are then combined without the metronome. As a last (and quite demanding) exercise the metronome is included as a third element beating quarter-notes*.


2 against 3.

EXERCISES 20 and 21, p. 36. After working through the exercises in 3 against 2 polyrhythms, the inversion, 2 against 3, will prove relatively easy to learn. In this case we perform the slower of the components, the duplet, in the upper part; thus it can be difficult to make the two notes completely even. Placing the second duplet note midway between the second and third notes of the underlying triplet will be a help at slower tempi. It is in fact more important to bear in mind that the duplet should be clearly *binary*, and that the triplet upper part in the 3 against

* This exercise can be executed only after the $\frac{5}{8}$ bar has been learnt. See my book "Ear-Training". Chester Music 1977, II (Rhythm Reading) pp. 25, 26, 52-54 and 71.

2 polyrhythm does not inadvertently influence the 2 against 3 pattern, which may cause the duplet to sound like this:  instead of like this: 

The following preliminary exercise should be practised to the accompaniment of the metronome; the upper part is sung, the middle part is clapped and the lower part tapped by the foot:



Repeat the last five measures at various more rapid tempi up to $\text{♩} =$ approx. 76; then work through exercises 20 and 21, p. 36.

2-PART EXERCISES 1-6, p. 58: six short introductory exercises for practising the two-part performance of the polyrhythms 3 against 2 and 2 against 3. The exercises are primarily intended for instrumental performance; they can be performed at the piano, but are equally intended for practise in polyrhythmic ensemble playing – performed by two stringed instruments or string groups (pizzicato is preferable, especially to begin with), a stringed instrument plus piano, etc. The part which contrasts with the basic meter should also be practised alone, while the metronome marks the contrasting cross-rhythm.

Then go on to exercises 1,2,3,4,5 and 6 in the volume “30 Polyrhythmic Etudes”*

* See preface

III. THE POLYRHYTHMS 3 AGAINST 4 AND 4 AGAINST 3

3 against 4

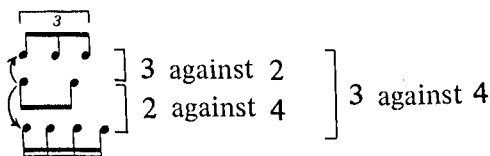
At first glance the 3 against 4 polyrhythm seems to represent a rather complicated juxtaposition; its two components, triplet and quadruplet, are so remote that we require a 12-fold division in order to see how they relate to each other:



But this purely mathematical proof is not much help in the practical performance of this polyrhythm; the 12-fold division, even at a moderate tempo, is a much too minute sub-division of the time scale. The frequently employed “interpolation” method: “the second triplet note a little after the second quadruplet note, the third triplet note a little before the fourth quadruplet note” cannot be recommended either, as it then becomes difficult to make both the contrasting figures even.

Nevertheless, the polyrhythm 3 against 4 and its inversion 4 against 3 (the latter type is the more common in the classical period) are both significant combinations. The frequent use of this pattern is doubtless due to the fact that in this and similar cases we are able to grasp simultaneously not only two contrasting *single rhythms*, as is the case with 3 against 2, but can also grasp two *rhythmic relationships*, which join together to form a new and perceptible pattern. In the case of 3 against 4 the two relationships 3 against 2 and 2 against 4 are combined. We will use this approach when learning this polyrhythm.

We are familiar with both relationships: 3 against 2, which we have just learnt, and 2 against 4, which poses no inherent difficulties. Thus we will use the duplet as the “mean proportional”:



In the following preliminary exercise, in which triplet and quadruplet divisions of the half-note are juxtaposed, we proceed to the 3 against 4 polyrhythm via 3 stages:



The following preliminary exercise is performed with the aid of the metronome and at a very moderate tempo (e.g. ♩ = 84) so that the relationship between the two contrasting rhythms emerges quite clearly; the upper part is sung, the middle part is clapped and the lower part is marked by the foot and the metronome. In the first stage the rhythmic part indicated with (a) is clapped as an accompaniment:

The musical score is in 4/4 time with a tempo marking of ♩ = 84. It consists of three systems of music. The first system features a vocal line (treble clef) and three rhythmic accompaniment parts labeled 'a.', 'b.', and 'c.'. Part 'a.' is a 4/4 rhythm with a quarter note followed by a triplet of eighth notes. Part 'b.' is a 4/4 rhythm with a quarter note followed by a triplet of eighth notes. Part 'c.' is a 4/4 rhythm with a quarter note followed by a triplet of eighth notes. The second system shows the vocal line and parts 'b.' and 'c.'. The third system shows the vocal line and parts 'b.' and 'c.'.

The two contrasting rhythms, triplet and quadruplet, are thus timed against the quarter-notes, which are marked by the foot and the metronome; by performing the rhythmic part (a) as accompaniment we can clearly perceive the eighth-notes as simply a sub-division of the second note of the duplet. Take care that the triplet does not drag and slow down the tempo – it may have a tendency to do so, while the eighth-notes in the quadruplet may tend to become too fast; the eighth-notes should be unhurried and exactly equal in length.

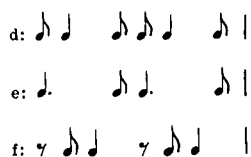
When the preliminary exercise, accompanied by the rhythmic part (a), has been mastered, exercises 3, 4, 5, 6, and 7 on p. 33 are sung through without a break to the accompaniment of the ostinato figure corresponding to (a) both with and without the foot and the metronome marking the basic pulse.

At the second stage the preliminary exercise is accompanied by ostinato-figure (b), and exercises 3-7 are sung through with the corresponding accompaniment figure.

Then, as the last stage, the full quadruplet figure (c) which is also used as accompaniment in exs. 3-7.

Then practise exs. 10 and 12 with the ostinato figures (a), (b) or (c) as an accompanying rhythm, thereby learning the polyrhythm 3 against 4 with the *triplet* figure in *derived* form.

Similarly, in order to learn the polyrhythm 3 against 4 with various derived forms of the *quadruplet*, exercises 3-7 are worked through to the accompaniment of ostinato figures such as:



Cases in which both triplet and quadruplet occur in derived forms can be tested, e.g. by performing exs. 10 and 12 to the accompaniment of the ostinato figure (d).

EXERCISES 22-27, p. 36. On the basis of the previous exercises, no. 22 poses no new problems. In exercises 23, 25, and 26, which are in eighth-note meter, the polyrhythm 3 against 4 occurs in its most common notational form, an eighth-note triplet in opposition to a figure in sixteenth-notes; as an introduction to this type ex. 8 can be performed to the accompaniment of one of the above mentioned ostinato figures in *diminished* form; as a preparation for ex. 24, in half-note meter, ex. 9 can be performed in a similar way, now with the ostinato figures in *augmented* form. Exercise 27 shows examples of polyrhythmic triplets extending across the bar-line.

EXERCISE 28, p. 38: the polyrhythm 3 against 4 in $\frac{7}{8}$ time*. In the accompanying rhythmic part the shifting accents are marked with a slight stress in measures 1 and 2: first, third and fifth eighth-note, in measure 3: first, third and sixth eighth-note, and so on. Initially the rhythmic part is practised alone on the basis of the metronome's *quarter-note* beats; check that the first beat in all the odd-numbered measures (measures 1, 3, 5 etc.) coincides with the metronome beat. Then the melody part and the accompaniment part are combined without the metronome.

2-PART EXERCISES 7 a-f and 8, p. 60: a group of short introductory exercises for practising the two-part performance of 3 against 4. On the first run-through the upper part is sung, the lower part is clapped and the metronome and foot mark the pulse beats at tempo 92 (in exercises 7 a-f eighth-notes, in ex. 8 quarter-notes). Then the lower parts in exs. 7 e and 8 are sung at the same tempo, while the upper parts are clapped. When the *pianist* transfers the exercises to his instrument, the metronome is also used as the fundamental element; the exercises are played without pedal, left hand slightly staccato, right hand slightly louder and more legato. If the polyrhythm causes difficulties initially, the right hand should be played first combined with the metronome (3 against 2); then the right and left hands are alternated, each playing one measure. the triplets in the right hand are thus introduced against the

* cf. my book *Ear Training*, II (Rhythm Reading) pp. 25ff, 54 and 71.

background of the metronome and the left hand. As it can be difficult at first to play a long series of triplets without them "running off the rails", the figure can be introduced initially with one triplet group, then two, and so on; in ex. 7a., for example, thus:

It is also important to work through the exercises in such a way that the parts are *each played by a separate performer*, e.g. the upper part by a violin, the lower part by the *piano* (doubling the part an octave lower), or by violin and cello; as already mentioned, it is best for string players to perform their parts *pizzicato*. The *singer* sings the upper part, while he or another player performs the lower part as a piano accompaniment. Polyrhythms distributed among two or more performers are in a way more demanding than if the same performer plays both components of the polyrhythm; both performers must perceive the overall structure, the mutual tension which the two component parts of the polyrhythm produce. The metronome is also used initially as a co-ordinating factor in ensemble exercises, as the "mean proportional" of the two contrasting rhythmic figures.

After working through this group of introductory exercises, nos. 9, 11, 12, 15, 16, 21, 22, 24 and 25 in the volume "*30 Polyrhythmic Etudes*" are practised for piano or instruments in ensemble.

4 against 3

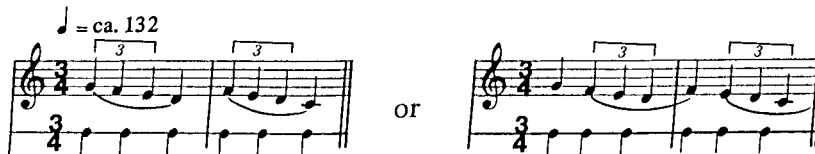
In the polyrhythm 4 against 3 the quadruplet, with its binary subdivisional type, is in the upper voice, while the rhythmic part or lower part represents the ternary subdivision, and it will most frequently be the quadruplet which will appear as the "foreign" rhythm and which must be timed against the triplets. On the basis of the 2 against 3 pattern, which we have already learnt, it is not difficult to reproduce the speed of the quadruplet, particularly when the figure is introduced as in the following example, where its character as a sub-divided duplet can be clearly perceived; the quadruplet movement begins halfway between the second and third beats of the bar. Sing the example accompanied by beating time audibly and supported by the metronome (tempo 120). Next time we adjust the metronome to two beats per bar

so that the meter and the metronome form 3 against 2; by maintaining the tempo the metronome's half-note duplets will thus correspond to tempo 80, thus $\frac{120 \times 2}{3} = 80$:



Perform the exercise at $\text{♩} = 108$ as well, and determine the tempo which the metronome should mark on the repetition of the exercise (half-note duplet).

When performing the rhythm 4 against 3 it should be borne in mind that an erroneous formation by analogy with the 3 against 2 pattern may make itself felt; if the last note of the quadruplet coincides with the last note of the ternary counterpart, then the 3 against 2 pattern will take over, as can easily happen at faster tempi and in connexion with a particular phrasing of the quadruplet, as in the following "false" 4 against 3 type:



The notes in the upper part may be precisely equal in length, but the polyrhythm may none the less be a "false" 4 against 3, as shown in the following example:



EXERCISES 29-35. p. 38. The exercises are accompanied by beating time audibly in the meter corresponding to the denominator of the time signature; they are practised both with and without the metronome. Note ex. 32 which is accompanied by beating $\frac{6}{8}$ -time. In exs. 32 and 33 the quadruplet figures are notated with their exact value (dotted notes, cf. chapter 1), in the other exercises the principle of "surplus value" is adhered to. The last of the exercises, no. 35, shows various derivations of the quadruplet figure.

A subtle rhythmic nuance appears when the polyrhythmic triplet corresponding to two pulse-beats is set beside the quadruplet which corresponds with three beats, as in the following example:



The speed of the two "foreign" rhythms seems to be almost the same, but if we compare the auxiliary rhythm we used to time the triplet: $\overline{\text{J}}\overline{\text{J}}\overline{\text{J}}$ with the auxiliary rhythm corresponding to the speed of the quadruplet: $\overline{\text{J}}\overline{\text{J}}\overline{\text{J}}\overline{\text{J}}$, it will be apparent that the quadruplet is very slightly slower than the preceding triplet. In a case like this we can also approach the problem by calculating the metronome figures for the two speeds and comparing them. As the tempo in this example is $\text{♩} = 84$, the half-note value must correspond to 42, and the single notes in the half-note triplet to $24 \times 3 = 126$. Similarly, the single notes in the ♩ quadruplet are equal to the metronome figure $\frac{84 \times 4}{3} = 112$. The quadruplet has the lower metronome marking and is therefore the slower.

Exercises 33 and 34 are particularly intended to develop ability in distinguishing clearly between the triplet and the quadruplet; apart from the difference in speed, the two polyrhythmic figures are different in nature, the triplet "soft", ternary, the quadruplet with its four-fold sub-division, "angular".

EXERCISE 36, p. 40. Sing ex. 30 at tempo 126 while the metronome marks *two* beats per measure, i.e. quarter-note duplets ($\overline{\text{J}} = 84$); on repetition the basic pulse is also clapped (eighth-notes). In ex. 36 duple, triple and quadruple sub-divisions are combined in the same way, the metronome is set at 100 to mark the eighth-notes, the middle part is clapped or tapped softly, while the upper part is sung. The exercise is repeated to the accompaniment of the metronome, which now marks the quarter-note value at tempo 58.

EXERCISES 37-41, p. 40. In exs. 37, 38 and 40 the triplets in the rhythm part occur in derived forms, and represent the contrasting rhythm in relation to the regular beat. Exercise 39 gives examples of polyrhythmic figures extending across the bar-line. In exercise 41 both the triplet and quadruplet occur in derived forms; initially the regular rhythm is marked with the foot. The five exercises are worked through first with the metronome marking the note-value given in the time signatures, and finally without.

2-PART EXERCISES 9-15, p. 61: 2-part exercises for learning the polyrhythm 4 against 3. Before performing the exercises instrumentally, the *upper voice* in exs. 12-15 is sung, while the rhythm of the lower part is clapped and tapped; similarly the *lower part* in exs. 10 and 12 is sung to the accompaniment of the rhythm of the upper part. During the preliminary exercises, and when the exercises are performed instrumentally, the metronome is used initially to mark the note-values indicated in the time signatures (in 9 and 10 first eighth-notes, then quarter-notes, in 11 quarter-notes etc.); check that there is no distortion of the tempo, and then omit the metronome. When the exercises are played on the piano or used in ensembles, the 4 against 3 pattern may seem rather more difficult than the polyrhythms already practised. Most frequently it is the erroneous analogy to the 3 against 2 polyrhythm which intrudes and causes the last note of the quadruplet to coincide with the last note of the triplet; in such cases it will be a help to give the

third note in each quadruplet group a slight accent, thus stressing the binary division of the figure. In ex. 15 all the four polyrhythmic types so far learnt occur.

Follow up by performing on the piano or in ensemble exs. 14, 18, 20, 26 or 27 in the volume "*30 Polyrhythmic Etudes*".

IV. POLYRHYTHMS DERIVED BY SUB-DIVIDING THE BASIC TYPES 3 AGAINST 2 AND 2 AGAINST 3

To this group belong, apart from 3 against 4 and 4 against 3 which have already been discussed, juxtapositions such as 6 against 2 or 4 (sextuplet = 2 + 2 + 2), the polyrhythmic 6 against 3 (sextuplet = 3 + 3) or forms in which the binary component, the quadruple division, is sub-divided, as in the combinations 8 against 3 and 8 against 6. The polyrhythms 6 against 2 and 6 against 3, may be called accentual polyrhythms, since in these cases the polyrhythmic relationship is only apparent if the sextuplet contrasts in accent with the contra part.

EXERCISE 42, p. 42: the polyrhythmic 6 against 2 and 6 against 4. This and the following exercises are performed while beating time audibly (quarter-note meter). Emphasize the accents of the sextuplets; in this case the sextuplets are sub-divisions of the quarter-note triplet, and the accents therefore fall on the *first*, *third* and *fifth* notes of the sextuplet, while the *fourth* note, which coincides with the basic pulse beat, is unstressed. The exercise is also performed to the accompaniment of the ostinato figures already employed: a. ♩ ♩ ♩ ♩ | b. ♩ ♩ ♩ ♩ | and c. ♩ ♩ ♩ ♩ |, so that the polyrhythmic combination is now equal to 6 against 4.

EXERCISE 43, p. 43: the same polyrhythms as in the previous exercise. Emphasize the changing stresses between, for example, measure 1 and measure 2 or measure 4 and measure 5; use the metronome to check that these changes in accent do not affect the basic tempo.

EXERCISE 44, p. 43: 9 against 2. In this case the juxtaposition can also be derived without difficulty from the basic 3 against 2 pattern.

EXERCISE 45, p. 43: 9 against 4. Perform the eight-note figures in the accompaniment part by beating time audibly. The polyrhythm 9 against 4 (measures 3, 5 and 7) is a characteristic example of how we are able to grasp rhythmic *relationships* in compound polyrhythms; the common denominator of the two components is a 36-fold division, which can obviously be of no assistance in performance; here the relationships 9 against 3 and 3 against 4 are combined, and the latter relationship is in itself a compound pattern.

EXERCISE 46, p. 43: 6 against 8. If the 6-fold division is a combination of 3 + 3, the polyrhythmic formation will correspond to twice 3 against 4; provided that the sextuplet is divided thus: 2 + 2 + 2; the timing of the polyrhythm is the same as 3 against 4, but the alteration of the stress must be clearly apparent in performance.

EXERCISE 47, p. 44: 4 against 6. The polyrhythms so far practised have been derivations of 3 against 2, while the present and following ones are of 2 against 3. Bring out the stresses in the accompaniment part.

EXERCISES 48-49. p.44: 8 against 3.

EXERCISE 50, p. 45: 8 against 6.

EXERCISE 51, p. 45: the polyrhythmic 6 against 3, a typical accentual polyrhythm, since the formation of the polyrhythm rests solely on the division of the sextuplet into 3 + 3. The combination is a derivation of the basic 2 against 3 form and its notation represents a case in which the "surplus-value" rule is not employed, since the sextuplets in the upper voice in the last half of the second measure and following should then be written in eighth-notes, despite the fact that the figure has the same speed as the sextuplet in the first measure (cf. chapter I).

2-PART EXERCISES 16-20, p. 64: exercises containing examples of the polyrhythms already dealt with. Ex. 16: the combination 3 against 2 in the first measures is continued as 6 against 4 because of the altered stress of the sextuplet; the polyrhythm is maintained throughout the exercise, except in the first two measures after the repeat sign. Ex. 17: *6 against 4* and *9 against 4*; besides these, 3 against 4 also occurs, from which the two subdivisional types are derived, as well as 3 against 2. Ex. 18: *4 against 6*; the quadruplet may tend to become too slow: begin by stressing the first, third and fifth note of the sextuplet — the 4 against 3 pattern is thus heard as an auxiliary rhythm. Ex. 19: *8 against 3*; the octuplet at the beginning of the third measure is timed by means of the quadruplets in the previous measure. Ex. 20: the *accentual* polyrhythm *6 against 3*; emphasize the ternary rhythm in the lower part; concerning the notation, cf. remarks on ex. 51.

V. QUINTUPLET AND SEPTUPLET POLYRHYTHMS

5 against 3

The 5 against 3 polyrhythm is a striking pattern which is relatively easy to learn, even if the common value into which both components divide is also rather remote in this case, i.e. a 15-fold sub-division. By sub-dividing the quintuplet we get the clearest impression of how the individual notes in each divisional figure relate to each other:



If we take the quintuplet figure as a basis and then introduce the three pulse beats, we can clearly hear the symmetry of the pattern: the second pulse-beat a triplet eighth-note *before* the third quintuplet note, the third pulse-beat a triplet eighth-note *after* the fourth quintuplet note:



Perform the example with quintuplet sub-divisions 2 + 3 and 3 + 2. The 5 against 3 juxtaposition is felt as a smoothly rounded, triplet-like polyrhythm related to 3 against 2 or 3 against 4, since triplet sub-divisions are used here in a similar way as auxiliary rhythms.

If a triplet corresponding to 4 beats, and a quintuplet corresponding to 3 beats, as in the following example, the speed of the quintuplet figure seems to be equal to double the speed of the triplet (example written in eighth-note meter):




but if we perform the quintuplet exactly twice as fast as the triplet, the fourth quintuplet note will coincide with the last pulse-beat in the $\frac{3}{8}$ measure; in this case we will perform a “false 5 against 3”, i.e. a binary triplet plus two sixteenth-notes:



The two patterns are very close to each other, so close that the “false” 5 against 3 can help us to find the speed of the quintuplet; we simply make the quintuplet figure a fraction faster than the triplet which corresponds to two beats, and the fourth quintuplet note thus falls into its correct place, immediately before the last pulse-beat in the $\frac{3}{8}$ measure.

In order to distinguish clearly between the two related rhythmic figures, the following example is performed to metronome accompaniment and while beating time audibly; repeat the last three measures, until the 5 against 3 pattern becomes perfectly clear and fluent:



EXERCISE 52, p. 46. The stresses of the quintuplet correspond in the first half of the exercise to 3 + 2, in the second half to 2 + 3; the stress in the 2 + 3 quintuplet can cause confusion with $\frac{3}{8}$  i.e. another kind of “false 5 against 3”. Accompany with the metronome and beat time audibly.

EXERCISE 53, p. 46. Beat time audibly to this and the following exercises, and use the metronome initially as well. The quintuplet occurs here preceded by sixteenth-note figures; note the changing stress of the quintuplet in measures 6 and 7.

EXERCISE 54, p. 46. This exercise contains the cross-rhythms ternary quadruplet and binary triplet; the quadruplet, with its “angular”, binary character is the slowest of the three; at tempo $\text{♩} = 72$ the quadruplet value is equal to $\frac{72 \times 4}{3} = 96$; the binary triplet is slightly faster, $\frac{72 \times 3}{2} = 108$, while the quintuplet is the most rapid: $\frac{72 \times 5}{3} = 120$. The three metronome indications give an impression of the subtle difference in speed between the three polyrhythmic formations. Note the vertical stress marks which indicate the accents of the quintuplet. The exercise should also be performed at tempo 96.

EXERCISE 55, p. 46: the two adjacent speeds, the ternary quintuplet and the binary triplet.

EXERCISE 56, p. 46: the same polyrhythms in $\frac{7}{8}$ time. Clap the rhythm of the accompaniment part or beat time to it audibly; emphasize the changing stresses of the $\frac{7}{8}$ measure with slight accents.

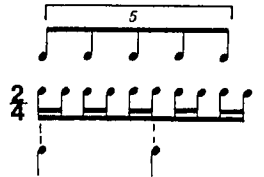
EXERCISE 57, p. 46. The quintuplet occurs here in several different derivations; c.f. the first example in this chapter, which shows how the individual notes relate to each other.

EXERCISE 58, p. 47. Here both the quintuplet figure and the ternary accompaniment rhythm occur in derived forms.

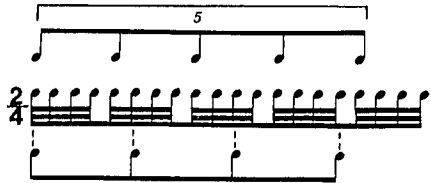
EXERCISE 59, p. 47. The polyrhythm 10 against 3 can be derived without difficulty from the basic 5 against 3 form.

5 against 2 and 5 against 4

The relationship between the two components in the 5 against 2 polyrhythm is quite simple; the second note of the duplet comes halfway between the third and fourth note of the quintuplet:



In the 5 against 4 combination the relationship is a little more complicated; this pattern is also clearly symmetrical, where the third quadruplet note forms a central axis:



In performance, the 5 against 2 polyrhythm, which appears relatively simple in mathematical terms, may become a bit tricky; the second half of the duplet may be delayed, so that it coincides with the fourth note of the quintuplet. In this connection it is of some importance where the accents fall within the quintuplet figure; if the quintuplet sub-divides into 3 + 2, the difficulty seems most pronounced; with the opposite sub-division, 2 + 3, on the other hand, the polyrhythm seems perfectly logical and easy to understand. Particularly in a case such as the following example, which to a certain extent is made more difficult by the initial rest in the quintuplet figures – the combination occurs in the first movement of Schumann's piano concerto – the polyrhythm can be learnt most readily by lightly stressing the right-hand notes marked with tenuto signs, corresponding to the 2 + 3 type of the quintuplet; when the figure is then performed *a tempo*, the auxiliary stresses are omitted.



EXERCISES 60-62, p. 47: 5 against 2. Accompany the exercise by beating time audibly; note the more difficult 3 + 2 sub-division of the quintuplet in ex. 62, measures 8-10.

EXERCISE 63, p. 48: preliminary exercise in the polyrhythm 5 against 4. The exercise is performed with the metronome. The 2 + 3 form of the quintuplet is maintained throughout the exercise and is emphasized by a slight accent on the third note in each quintuplet group. In performance, be careful that the fourth note in the underlying quadruplet does not coincide with the last note of the quintuplet. Then repeat ex. 60 with quarter-notes as the accompanying rhythm, together with exs. 61 and 62, in which eighth-notes are clapped as an accompaniment.

EXERCISES 64-67, p. 49: 5 against 4. In ex. 65 the quadruplet accompaniment part occurs with the insertion of rests, while in ex. 66 the quintuplet occurs in derived forms. Ex. 67: here the 3-meter quintuplet and the 4-meter quintuplet follow each other in measures 1 and 2 — respectively the fastest and the slowest cross-rhythms in the exercise; from measure 5 and thereafter the cross-rhythms decrease more and more in speed, from the 2-meter triplet through the 3-meter quadruplet to the 4-meter quintuplet; check the accuracy with the aid of the metronome.

EXERCISE 68, p. 50. The polyrhythm 10 against 4 can be derived without difficulty from the combination 5 against 4; the unstressed sixth note of the 10-fold division coincides with the third pulse-beat of the measure, and the timing thus corresponds to 5 against 2, but with a change in stress.

EXERCISE 69, p. 50: an exercise which shows that a juxtaposition of three mutually contrasting sub-divisional types is not impossibly difficult. In the first half of the exercise the metronome marks the *duplet division*, while the rhythmic part marks the *triplet division* and the vocal upper part the *quintuplet division*: in the last half, three adjacent divisions are combined, a triplet division, a quadruplet division and a quintuplet division; in the last, rather more difficult combination (from measure 7 and following) the rhythmic part forms 4 against 3 in relation to the metronome, a pattern which is in itself a compound pattern; it may be a help to give a slight accent to the third note in the sixteenth-note groups of the rhythmic part.

7 against 2 and 7 against 4

The polyrhythms dealt with so far have consisted of various combinations of the first three basic forms: duplet division, triplet division and quintuplet division. With polyrhythms containing septuplets the fourth primary subdivision, is also included, the last in the series which it is worth considering. If we proceed further to 11- or 13-fold divisions — the basic forms, as mentioned above, follow the prime series — we find that these divisional types are so inherently unstable that they cannot form part of real polyrhythmic structures; they can occur in connexion with rapid roulades, or in a context which makes it obvious that an improvisational, non-rhythmic performance is in fact intended.

The simplest of the septuplet polyrhythms, 7 against 2, poses no great difficulties after the 5 against 2 combination has been learnt; the second note of the duplet falls halfway between the fourth and fifth note of the septuplet; the polyrhythm 7 against 2 seems to be easiest to perform with the sub-division 2 + 3 + 2. See the 2-part exercise no. 28, p. 70.

Let us imagine a 4-meter measure occupied by a septuplet; the figure then forms 7 against 4 in relation to the basic meter. The timing of the septuplet against the four beats seems quite complicated at first, but the difficulty is actually not so great; in the same way as the quintuplet in 5 against 3 could be derived from the two-beat triplet, the four-beat septuplet can be obtained even more easily from the three-beat quintuplet:

The image shows two systems of musical notation in 4/8 time. The first system consists of two measures. The first measure contains a quintuplet of five eighth notes, with a bracket above it labeled '5'. The second measure contains a septuplet of seven eighth notes, with a bracket above it labeled '7'. The tempo is indicated as quarter note = 96. The second system is similar, but the first measure contains a quintuplet of five eighth notes (bracketed '5') and the second measure contains a septuplet of seven eighth notes (bracketed '7').

Measures 2 and 5 are practically indistinguishable in terms of rhythmic structure; while the sixth note in measure 2 coincides with the fourth pulse beat, in the fifth measure the sixth septuplet note falls $1/7$ of the meter value *before* the fourth beat; the 4-beat septuplet is thus only marginally faster than the 3-beat quintuplet. At the given tempo, $\text{♩} = 96$, the metronome marking of the quintuplet will be equal to $\frac{26 \times 5}{3} = 160$, the septuplet to: $\frac{26 \times 7}{4} = 168$; these two figures are adjacent markings on the metronome scale and relation between them thus corresponds to the smallest difference in degree normally used in metronome indications.

In the same way as the human ear, when confronted with intonation or interval sizes, has a certain level of tolerance (which thus enables us to accept the system of tempered tuning), there is also a similar kind of tolerance in connection with rhythmic phenomena; by analogy, therefore, we can accept the 7 against 4 polyrhythm as a pattern which is as plausible and comprehensible as 5 against 3, despite the fact that the combination 7 against 4 is extremely complicated mathematically:

The image shows a musical notation for a 7 against 4 polyrhythm. The top staff shows a septuplet of seven eighth notes, with a bracket above it labeled '7'. The bottom staff shows a 4-beat septuplet of seven eighth notes, with a bracket above it labeled '7'. The tempo is indicated as quarter note = 96.

We can at the same time deduce that apparently complicated combinations *can* "fit in" as entirely plausible patterns, so long as they are backed up by closely related analogous formations; in this respect possibilities increase as we proceed towards more complex types of polyrhythms.

EXERCISES 70-73, p. 51: exercises in 7 against 4. Beat time audibly. The minimal difference between the 3-beat quintuplet and the 4-beat septuplet (e.g. ex. 70, measures 5-8) can be most clearly perceived in the aural character of the two polyrhythms, 5 against 3 which is rounded, triplet-like, and 7 against 4 which is binary in nature. Note in ex. 71 the rather unusual notation in sixteenth-note meter. In ex. 72 the quarter-note septuplets are given square brackets which indicate the septuplet division. Ex. 73: the polyrhythm 7 against 4 is sufficiently stable to allow various derivatives of the septuplet to be performed without great difficulty.

7 against 3

The following diagram shows how the individual notes in the two sub-divisions relate to each other:



The 7 against 3 combination occurs most frequently at relatively rapid tempi. The polyrhythm is easiest to perform when the septuplet is divided 2 + 2 + 3; however, if the duplet grouping comes last, as in 3 + 2 + 2 or 2 + 3 + 2, the sixth note will, particularly in more rapid tempi, tend to coincide with the third note in the underlying triplet, as in the following passage from Beethoven's G major piano concerto. If we want to perform the passage as a "genuine 7 against 3" (which is probably not absolutely necessary), we must as far as possible avoid accenting the sixth note of the septuplet:

(Allegro moderato)

EXERCISES 74-76, p. 52. Beat time audibly to these exercises. The septuplet in exercise 74 (measure 5) can readily be timed by means of the figures in measures 1-3. Pay attention to the perfect evenness of the septuplet figures – all notes of equal length; the three-note grouping at the end of the figure may tend to become too fast. Ex. 75: the nonuplet in the polyrhythm 9 against 4, a combination which has already been practised (see ex. 45 and the two-part ex. no. 17, p. 67), has almost the same "speed" as the 3-beat septuplet; at the given tempo (132) the metronome markings for the two figures are 297 and 308 respectively. These markings do not appear on the metronome scale, but they show that the septuplet is a fraction faster – a distinction which is barely perceptible. Use the metronome as a check. Ex. 76: the septuplet appears here, where the tempo is slower, in its three different sub-divisional types; the exercise also contains the 3-beat quintuplet and the 2-beat triplet.

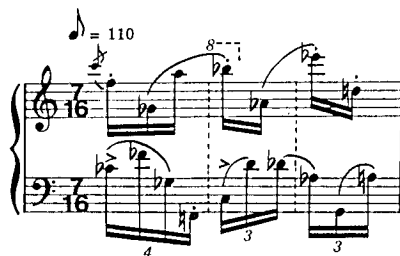
EXERCISE 77, p. 53: a combination exercise in which the polyrhythms 7 against 4 and 7 against 3 are juxtaposed. The metronome marks the triplets, the rhythmic part the quadruplets and the vocal upper part the septuplets.

2-PART EXERCISES 21-32, p. 67. *Exs. 21-23: 5 against 3.* In exercises 21 and 23 the quintuplet figure corresponds to 3 + 2, in 22 to 2 + 3. *No. 24: 5 against 2.* Here the two quintriplet types alternate; emphasize the 2 + 3 division of the quintuplet in connexion with the 5 against 2 polyrhythm. *No. 25.* When the quintuplet figure begins with a rest, as in measures 1, 3 and 5, it is particularly important to emphasize the 2 + 3 division of the quintuplet, in the present case by giving the second note a slight accent. *No 26: 5 against 4.* In this rather more difficult combination it is also a considerable help to emphasize the third note of the quintuplet in compliance with the division into 2 + 3. *Ex. 27.* The polyrhythm 5 against 4 occurs here with the underlying quadruplet in derived forms. *No. 28: 7 against 2.* Emphasize the septuplet division 2 + 3 + 2. *Nos. 29 and 30: 7 against 4.* Note the changing sub-divisions of the septuplet. *Nos. 31 and 32: 7 against 3.* Where the septuplet moves at a rapid tempo, as in ex. 31, the principal difficulty is to make the seven notes of the septuplet perfectly even, without the concluding three-note grouping becoming faster; note the changing secondary accents in the septuplets in ex. 32. Then perform nos. 7, 8, 10, 17, 23 and 29 in the volume "30 Polyrythmic Etudes" for piano or instrumental ensembles.

VI. POLYRHYTHMS IN QUINTUPLE AND SEPTUPLE METER AND IN ASSOCIATION WITH CHANGING METERS

Having dealt in the preceding chapters with rhythms in which quintuplet and septuplet figures occurred as cross-rhythms against duplet and triplet groupings, in this last section we will turn things upside down, so that 5- or 7-fold divisions now form the basis for polyrhythmic combinations. We will thus be concerned with the analysis of certain forms which occur particularly in modern music; another characteristic feature of this style is the frequently changing meter, which further highlights the analogy with the metrical forms of prose.

Irregular time signatures are a potentially fruitful source of polyrhythmic formations; apart from the cases in which a polyrhythm is built up over a full 5- or 7- meter measure, parts of the measure – its components consisting of binary or ternary groupings – may also form the basis of contrasting cross-rhythms, as demonstrated in the following extract from Stravinsky's *"Movements"*, 1959 (measure 17):



The tempo indicated ($\text{♩} = 110$) makes this passage extremely difficult to perform*; but in other respects the polyrhythms which occur – 3 against 4 and 2 against 3 – are types with which we are already familiar and which at a slower tempo would seem more immediately comprehensible (see exs. 19, 28 and 56 as well as *"30 Polyrhythmic Etudes"* nos. 8, 13, 16, 17 and 28, in which parts of 5- and 7-meter measures act as the basis for various polyrhythmic forms).

However, we are left with cases in which a complete quintuplet or septuplet grouping in the lower part is combined with a contrasting cross-rhythm in the upper part, types such as 7, 8 or 9 against 5, and 4, 3 or 2 against 5, as well as 4 against 7 or 5 against 7. These forms will be dealt with below.

* Stravinsky said ". . . its rhythmic language is also the most advanced I have so far employed" about *"Movements"* in a conversation with Craft.

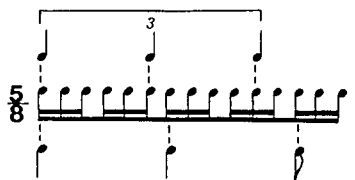
The first group of these polyrhythmic combinations, 7, 8 or 9 against 5, appears at first glance to be complicated and difficult to perform; but, as mentioned previously, these complex polyrhythms can often be performed on the basis of their analogy with combinations which are structurally simpler and with which we are already familiar.

EXERCISE 78, p. 54: *8 against 5*. The octuplets in measures 8, 10-12 and 14 occupy a full $\frac{5}{8}$ measure in relation to the basic meter; the timing of the 5-meter octuplet is practically identical with that of the ternary quintuplet (measure 2); at the given tempo, 96, the metronome marking of the ternary quintuplet will be equal to 160 and the 5-meter octuplet to 153.6, which shows that the octuplet is the slower of the two, but only to so small an extent that the difference is very nearly imperceptible; the difference between the two figures will be seen to be less than the difference between two adjacent markings on the metronome (160 and 152). In measure 10 foll. where the octuplet assumes the form of a sub-divided duplet, the 8 against 5 combination is particularly easy to perform, as the accented first note in the second duplet grouping falls halfway between the third and fourth beats in the 5-meter measure. The five pulse-beats in the rhythmic accompaniment can be marked by tapping the five fingers of one hand one after the other.

EXERCISE 79, p. 54: *7 against 5*. In measure 1 a ternary meter quadruplet is followed by a binary triplet; as already mentioned, the two figures are close to each other in terms of speed (cf. remarks on exs. 29-35); at tempo 96 the speeds correspond to mm. = 128 and 144 respectively; the 5-meter septuplet (measures 5 and 7), however, has a speed which is more or less halfway between the two (mm. = 134.4). The septuplet in measure 5, therefore, should be a fraction faster than the quadruplet in the preceding measure.

EXERCISE 80, p. 54: *9 against 5*. We have already seen how the 4-meter septuplet could be derived from the ternary quintuplet; the distance between the 4-meter septuplet and the 5-meter nonuplet is even smaller; at tempo 96 the two speeds are equal to mm. 168 and 172.6, a difference which corresponds to only about half the distance between the two neighbouring marks on the metronome scale. Finally, perform exs. 78, 79 and 80 without a break.

EXERCISE 81a, p. 54: preliminary exercise in the polyrhythm 3 against 5. Like 5 against 3, its inversion is also a relatively stable pattern; however, as this polyrhythm occurs frequently at quite rapid tempi, it is advisable to practise the 5-meter triplet by beating $\frac{5}{8}$ time marking audibly only the first, third and fifth pulse beats (a $\frac{3}{4}$ measure with a short 3rd beat, cf. the comment to ex. 19). The following diagram shows how the three notes of the triplet relate to the three marked pulse-beats in the 5-meter measure (note that the short 5th beat does not fall exactly halfway between the last note of the triplet and the following first beat, but a fraction earlier):



On the first run-through of ex. 81a the upper part is sung while the rhythm part is beaten audibly as explained; the metronome is then set at tempo 100 and combined with the rhythmic part, and finally the vocal part is added. On the final run-through the rhythmic part is omitted in measures 7-10, thus producing the polyrhythm *6 against 5* in measures 7-8.

EXERCISE 81b, p. 55 is worked through on the same lines as 81a; here the triplet figure, which occupies a $\frac{5}{8}$ measure (measures 3 and 7) occurs alongside the slightly faster triplet which occupies a $\frac{2}{4}$ measure (measures 5 and 6).

EXERCISE 82, p. 55. The polyrhythm *3 against 5* occurs here in a context of changing time signatures; beat time audibly as an accompaniment.

EXERCISES 83-84, p. 56: the same two triplet figures as in ex. 81b. Note the diminution of the time signature in ex. 84; the 1st and 5th measures are accompanied by beating $\frac{5}{16}$ time audibly.

EXERCISE 85, p. 56: *4 against 5* and *2 against 5*. It is not difficult to time the 5-meter quadruplet in measure 4 against the 5-meter triplet in measure 1. In the $\frac{6}{8}$ measures, beat time as for a 4-meter bar with short second and fourth beats.

EXERCISE 81c, p. 55: preliminary exercise for the polyrhythm *4 against 7*. Work through the exercise in the same way as exs. 81a and b; the 7-meter quadruplet in measure 5 is a fraction slower than the 5-meter triplet in measure 3. In the $\frac{7}{8}$ measures, beat time as for a 4-meter measure with a short fourth beat.

EXERCISE 86, p. 57: *4 against 7*. Beat time audibly as an accompaniment.

And finally the following test:

Perform exs. 82, 83, 85 and 86 while the metronome marks quarter-notes at tempo 100. In half the irregular measures ($\frac{5}{8}$ and $\frac{7}{8}$) the accompanying rhythmic part will syncopate with the metronome beat, and in each of the exercises its final note will coincide with the metronome beat. Exercise 84 is performed in a similar way, with the metronome marking eighth-notes at tempo 100; and as a last test *Exercise 87, p. 57*, which is also performed with the metronome; here it marks quarter-notes at tempo 88.

2-PART EXERCISES NOS. 33-26, p. 72: four examples of polyrhythms belonging to the last group. Exercise 33: *3 against 5*. In performance ensure that the second quarter-note in the upper part falls shortly *before* the third note in the underlying quintuplet. Exercise 34: *9 against 5*. The close analogy between this polyrhythm and

7 against 4 and 5 against 3 makes it quite easy to time the nonuplet. Exercise 35: 5 against 7. The quintuplet in the upper part may easily assume the form of a 4-beat triplet followed by a 3 beat duplet; at tempo $\text{♩} = 112$ the 4-beat triplet will be equal to mm. $\frac{112 \times 2 \times 3}{4} = 168$, the 7-beat quintuplet to $\frac{112 \times 2 \times 5}{7} = 160$; the quintuplet is thus only very slightly slower, and its fourth note will therefore come a fraction after the sixth note in the underlying septuplet.* In exs. 33, 35 and 36 perform the upper part *legato* and the lower part *staccato*, in ex. 34 both parts *legato*.

Finally, perform nos. 13, 19, 28 and 30 of "30 Polyhythmic Etudes" on the piano or in ensemble.

* Exercise 36: 4 against 7. Check that the fourth note in the quadruplet falls a fraction after the sixth beat in the 7-meter measure.

I

$\text{♩} = 92$

1 $\text{♩} = 92$

2

3

4

5


6

7

8

9

10 

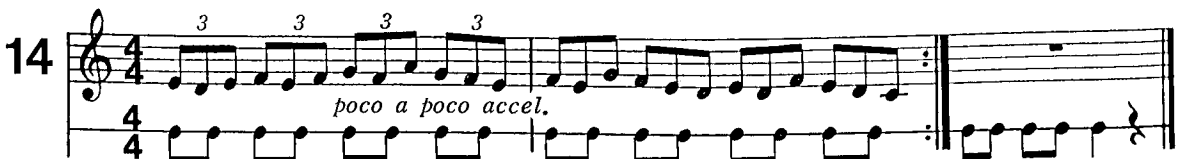
 11 



12 

 13 



14 
poco a poco accel.

15 
♩ = 72



16 $\text{♩} = 104$ 3

17 $\text{♩} = 80$ 3

18 $\text{♩} = 80$ 3

The musical score consists of three systems, each with a treble and bass staff. System 16 (measures 16-17) is in 4/8 time with a tempo of 104. It features a treble staff with eighth-note patterns and triplets, and a bass staff with eighth-note accompaniment. System 17 (measures 17-18) is in 4/8 time with a tempo of 80. It includes a treble staff with eighth-note patterns and triplets, and a bass staff with eighth-note accompaniment. System 18 (measures 18-19) is in 3/2 time with a tempo of 80. It features a treble staff with quarter-note patterns and triplets, and a bass staff with quarter-note accompaniment. The score includes various musical notations such as slurs, triplets, and dynamic markings.

36

♩ = ca. 92

19

20

21

♩ = 60

22

♩ = 76

Musical notation for the first system, featuring a treble clef and a bass clef. The melody consists of eighth notes with triplet markings above them. The bass line consists of eighth notes.

23

$\text{♩} = 84$

Musical notation for the second system of exercise 23, continuing the melody and bass line with triplet markings.

24

$\text{♩} = 76$

a. $\frac{4}{2}$

b. $\frac{2}{2}$

25

$\text{♩} = 84$

Musical notation for the second system of exercise 25, continuing the melody and bass line with triplet markings.

26

$\text{♩} = 80$

Musical notation for the second system of exercise 26, continuing the melody and bass line with triplet markings.

27 $\text{♩} = 76$

Musical notation for measure 27 in 4/4 time. The tempo is marked as quarter note = 76. The melody consists of three triplet groups of eighth notes. The bass line consists of eighth notes.

28 $\text{♩} = 92$

Musical notation for measure 28 in 7/8 time. The tempo is marked as quarter note = 92. The melody features a triplet of eighth notes. The bass line consists of eighth notes.

Musical notation for measure 29 in 4/4 time. The melody features two triplet groups of eighth notes. The bass line consists of eighth notes.

Musical notation for measure 30 in 5/8 time. The melody features a triplet of eighth notes. The bass line consists of eighth notes.

Musical notation for measure 31 in 7/8 time. The melody features two triplet groups of eighth notes. The bass line consists of eighth notes.

29 $\text{♩} = 112$

Musical notation for measure 29 in 3/4 time. The tempo is marked as quarter note = 112. The melody features two quartet groups of eighth notes. The bass line consists of eighth notes.

30

Musical notation for measure 30 in 3/8 time. The melody features two quartet groups of eighth notes. The bass line consists of eighth notes.

31

Musical notation for measure 31 in 3/2 time. The melody features two quartet groups of eighth notes. The bass line consists of eighth notes.

Musical staff with treble clef, showing a sequence of notes with a 4-measure slur, another 4-measure slur, and a 3-measure slur.

32

33

Musical staff with treble clef, showing a sequence of eighth notes with slurs and triplets.

Musical staff with treble clef, showing a sequence of eighth notes with slurs and a triplet.

34

35

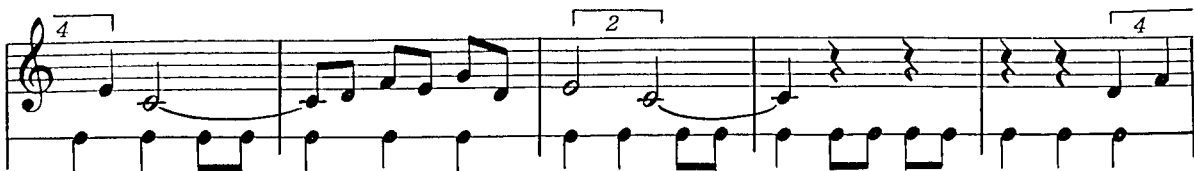
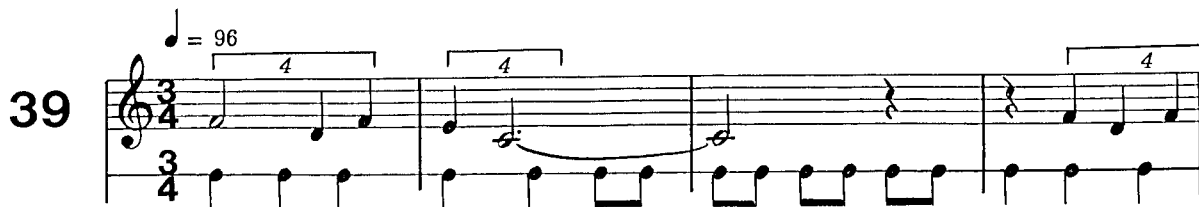
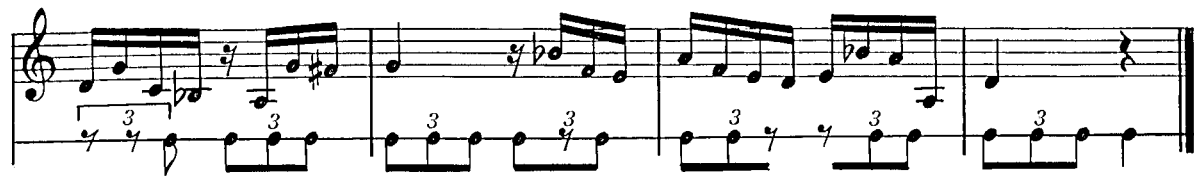
Musical staff with treble clef, showing a sequence of eighth notes with slurs and a 2-measure slur.

Musical staff with treble clef, showing a sequence of eighth notes with slurs and a 2-measure slur.

36 $\text{♩} = 100, \text{♩} = 58$

Met.

37 $\text{♩} = 80 - 88$



40 $\text{♩} = 84$

Musical notation for exercise 40, measures 1-4. Treble clef, 3/8 time signature. Features eighth notes and triplets.

Musical notation for exercise 40, measures 5-8. Treble clef, 3/8 time signature. Features eighth notes and triplets.

41 $\text{♩} = 60$

Musical notation for exercise 41, measures 1-4. Treble clef, 3/4 time signature. Features eighth notes and triplets.

Musical notation for exercise 41, measures 5-8. Treble clef, 3/4 time signature. Features eighth notes and triplets.

Musical notation for exercise 41, measures 9-12. Treble clef, 3/4 time signature. Features eighth notes and triplets.

42 $\text{♩} = 80$

Musical notation for exercise 42, measures 1-2. Treble clef, 4/4 time signature. Features quarter notes and triplets.

Musical notation for exercise 42, measures 3-6. Treble clef, 4/4 time signature. Features quarter notes and triplets.

43 $\text{♩} = 76$

44 $\text{♩} = 69-80$

45 $\text{♩} = 100$

46 $\text{♩} = 88$

47

$\text{♩} = 92$

48

$\text{♩} = 88$

49 $\text{♩} = 76$

Musical score for exercise 49, measures 1-4. Treble clef, key signature of one sharp (F#). Measure 1 has a whole rest. Measures 2-4 contain eighth notes with triplets in the bass line.

Musical score for exercise 49, measures 5-7. Treble clef, key signature of one sharp (F#). Measure 5 has a triplet in the bass line. Measure 6 has a slur over the treble line. Measure 7 has a whole note in the treble line.

50 $\text{♩} = 126$

Musical score for exercise 50, measures 1-2. Treble clef, key signature of one flat (Bb). Measure 1 has a whole note. Measure 2 has a triplet in the bass line.

Musical score for exercise 50, measures 3-4. Treble clef, key signature of one flat (Bb). Measure 3 has a 3/4 time signature change. Measure 4 has a slur over the treble line with a '2' above it.

Musical score for exercise 50, measures 5-7. Treble clef, key signature of one flat (Bb). Measure 5 has a slur over the treble line with a '2' above it. Measure 6 has a slur over the treble line with a '2' above it. Measure 7 has a whole note in the treble line.

51 $\text{♩} = 132$

Musical score for exercise 51, measures 1-3. Treble clef, key signature of one flat (Bb). Measure 1 has a whole note. Measures 2-3 have eighth notes with slurs and accents.

Musical score for exercise 51, measures 4-6. Treble clef, key signature of one flat (Bb). Measure 4 has a slur over the treble line. Measure 5 has a slur over the treble line. Measure 6 has a whole note in the treble line.

52 $\text{♩} = 96$

53 $\text{♩} = 88$

54 $\text{♩} = 72, \text{♩} = 96$

55 $\text{♩} = 100$

56 $\text{♩} = 96, \text{♩} = 108$

57 $\text{♩} = 120$

Musical notation for the first system of exercises. The treble clef contains five-measure phrases, each marked with a bracket and the number '5'. The bass clef contains a steady eighth-note accompaniment.

Musical notation for the second system of exercises. The treble clef features a grace note followed by two five-measure phrases. The bass clef continues with the eighth-note accompaniment. The system concludes with a double bar line.

58

$\text{♩} = 92$

Musical notation for exercise 58. The tempo is marked as quarter note = 92. The piece is in 3/8 time. The treble clef contains five-measure phrases, and the bass clef features a complex eighth-note accompaniment.

Musical notation for the third system of exercises. The treble clef includes a triplet and several five-measure phrases. The bass clef continues with the eighth-note accompaniment.

$\text{♩} = 80$

59

Musical notation for the fourth system of exercises. The tempo is marked as quarter note = 80. Exercise 59 is in 3/8 time and features a five-measure phrase in the treble clef and eighth-note accompaniment in the bass clef.

Musical notation for the fifth system of exercises. The treble clef contains five-measure phrases, and the bass clef continues with the eighth-note accompaniment.

60

$\text{♩} = 66$

Musical notation for exercise 60. The tempo is marked as quarter note = 66. The piece is in 2/2 time. The treble clef contains triplet eighth-note phrases, and the bass clef features a steady half-note accompaniment.

Musical notation for the sixth system of exercises. The treble clef contains five-measure phrases, and the bass clef continues with the half-note accompaniment.

61 $\text{♩} = 72$

62 $\text{♩} = 69$

63 $\text{♩} = 76$

Musical notation for the first system, featuring a treble and bass staff with a 4/4 time signature. The melody consists of five-measure phrases, each marked with a '5' above the notes. The bass line provides a steady accompaniment of eighth notes.

64

Musical notation for the second system, starting with a tempo marking of quarter note = 76. It includes a treble and bass staff with a 4/4 time signature. The melody features triplets and five-measure phrases. The bass line continues with eighth notes.

Musical notation for the third system, featuring a treble and bass staff with a 4/4 time signature. The melody consists of five-measure phrases, each marked with a '5' above the notes. The bass line provides a steady accompaniment of eighth notes.

Musical notation for the fourth system, featuring a treble and bass staff with a 4/4 time signature. The melody consists of five-measure phrases, each marked with a '5' above the notes. The bass line provides a steady accompaniment of eighth notes.

65

Musical notation for the fifth system, starting with a tempo marking of quarter note = 69. It includes a treble and bass staff with a 4/8 time signature. The melody features five-measure phrases. The bass line continues with eighth notes.

Musical notation for the sixth system, featuring a treble and bass staff with a 4/8 time signature. The melody consists of five-measure phrases, each marked with a '5' above the notes. The bass line provides a steady accompaniment of eighth notes.

Musical notation for the seventh system, featuring a treble and bass staff with a 4/8 time signature. The melody features triplets and five-measure phrases. The bass line continues with eighth notes.

66 $\text{♩} = 112 - 132$

67 $\text{♩} = 104$

68 $\text{♩} = 92$

69 $\text{♩} = 96$

Metr.

70

71

74 $\text{♩} = 104$

Musical score for exercise 74, measures 1-4. It features a treble and bass staff in 3/8 time. The melody includes a quintuplet in measure 3. The bass line consists of eighth notes.

75 $\text{♩} = 132$

Musical score for exercise 75, measures 1-4. It features a treble and bass staff in 4/16 time. The melody includes triplets and a quintuplet. The bass line consists of eighth notes.

76 $\text{♩} = 92$

Musical score for exercise 76, measures 1-4. It features a treble and bass staff in 3/8 time. The melody includes a quintuplet and a septuplet. The bass line consists of eighth notes.

The first system of music consists of two staves. The upper staff is in treble clef and contains a sequence of notes with various rhythmic values and fingerings: a quarter rest (marked with a 'y'), an eighth-note triplet (marked with a '7'), a quarter rest (marked with a 'y'), another eighth-note triplet (marked with a '7'), a quarter note (marked with a '3'), and a quarter-note quintuplet (marked with a '5'). The lower staff is in bass clef and provides a steady accompaniment of eighth notes.

The second system begins with a treble staff containing a double bar line. To the right of this staff is the number '77'. Below the treble staff is the word 'Metr.' followed by a metronome symbol. To the right of the metronome is a tempo indicator '♩ = 96'. The system continues with two staves: the upper staff has a whole rest, and the lower staff has a steady eighth-note accompaniment. The system concludes with two measures of a four-note chord in the lower staff, each marked with a '4'.

The third system consists of two staves. The upper staff features a seven-note scale starting on a quarter note, marked with a '7'. The lower staff provides a steady accompaniment of eighth notes, with a '4' indicating a four-fingered pattern.

The fourth system consists of two staves. The upper staff features a seven-note scale starting on a quarter note, marked with a '7'. The lower staff provides a steady accompaniment of eighth notes, with a '4' indicating a four-fingered pattern.

The fifth system consists of two staves. The upper staff features a seven-note scale starting on a quarter note, marked with a '7'. The lower staff provides a steady accompaniment of eighth notes, with a '4' indicating a four-fingered pattern.

The sixth system consists of two staves. The upper staff features a seven-note scale starting on a quarter note, marked with a '7'. The lower staff provides a steady accompaniment of eighth notes, with a '4' indicating a four-fingered pattern.

78

♩ = 96

79

♩ = 96

80

♩ = 96

81

a) ♩ = 100

Metr.

First system of musical notation. Treble clef, one flat key signature, 2/4 time signature. Melody: quarter note G4, eighth note triplet (A4, B4, C5), quarter note B4, quarter note A4, quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4.

Second system of musical notation. Treble clef, one flat key signature, 2/4 time signature. Melody: quarter note D4, quarter note C4, quarter note B3, quarter note A3, quarter note G3. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4.

Third system of musical notation, labeled 'b)'. Treble clef, 5/8 time signature, one flat key signature. Melody: quarter note G4, eighth note triplet (A4, B4, C5), quarter note B4, quarter note A4, quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4. Time signature changes to 2/4.

Fourth system of musical notation. Treble clef, 2/4 time signature, one flat key signature. Melody: quarter note G4, eighth note triplet (A4, B4, C5), quarter note B4, quarter note A4, quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4. Time signature changes to 5/8.

Fifth system of musical notation, labeled 'c)'. Treble clef, 5/8 time signature, one flat key signature. Melody: quarter note G4, eighth note triplet (A4, B4, C5), quarter note B4, quarter note A4, quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4. Time signature changes to 4/4.

Sixth system of musical notation. Treble clef, 4/4 time signature, one flat key signature. Melody: quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4.

Seventh system of musical notation, labeled '82'. Treble clef, 5/8 time signature, one flat key signature. Melody: quarter note G4, eighth note triplet (A4, B4, C5), quarter note B4, quarter note A4, quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4. Time signature changes to 3/4.

Eighth system of musical notation. Treble clef, 5/8 time signature, one flat key signature. Melody: quarter note G4, eighth note triplet (A4, B4, C5), quarter note B4, quarter note A4, quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4. Time signature changes to 3/4.

Ninth system of musical notation. Treble clef, 5/8 time signature, one flat key signature. Melody: quarter note G4, eighth note triplet (A4, B4, C5), quarter note B4, quarter note A4, quarter note G4, quarter note F4, quarter note E4, quarter note D4. Bass line: eighth notes G3, A3, B3, C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4. Time signature changes to 3/4.

83 $\text{♩} = 100$

5/8 5/8 3/4 2/4 2/4 5/8 5/8

5/8 5/8 2/4 2/4 5/8 5/8 4/4 4/4

4/4 4/4

84 $\text{♩} = 100$

10/16 10/16 4/8 4/8

4/8 3/8 10/16 3/8 3/8

85 $\text{♩} = 100$

5/8 5/8 3/4 5/8 5/8

3/4 3/4 6/8 4/4 5/8 5/8

5/8 3/4 3/4 5/8 5/8

86 $\text{♩} = 100$

Musical score for exercise 86, measures 1-4. The piece is in 7/8 time with a tempo of 100. The melody is written in the treble clef and includes triplets and a 5-measure phrase. The bass line is in the bass clef and consists of eighth notes.

87 $\text{♩} = 88$

Musical score for exercise 87, measures 1-4. The piece is in 3/4 time with a tempo of 88. The melody is written in the treble clef and includes quintuplets and triplet phrases. The bass line is in the bass clef and consists of eighth notes.

II

1

$\text{♩} = 120, \text{♩} = 60$

2

$\text{♩} = 104$

3

3

$\text{♩} = 144$

4

$\text{♩} = 72$

The first system consists of two staves. The upper staff contains a continuous eighth-note melody. The lower staff contains a similar eighth-note accompaniment, with a slur under the final two measures.

5

The second system is marked with a tempo of quarter note = 112. It features a treble staff with a melodic line and a bass staff with triplet accompaniment. A large number '5' is placed to the left of the system.

The third system continues the piece with triplet accompaniment in the bass staff. The treble staff has a melodic line with some rests.

The fourth system includes a tempo marking of quarter note = 84 and a section marker '6'. It features a treble staff and a bass staff with triplet accompaniment. The system concludes with a double bar line.

The fifth system features a treble staff and a bass staff with triplet accompaniment. The treble staff has a melodic line with some rests. The system concludes with a double bar line.

a) $\text{♩} = 96$

7

b)

c)

d)

e)

8

$\text{♩} = 88$

9

$\text{♩} = 100, \text{♩} = 54$

3

3

10

$\text{♩} = 126, \text{♩} = 66$

3

62

11

$\text{♩} = 72$

62 63 64 65

66 67 68 69

12

$\text{♩} = 60$

70 71 72 73

74 75 76 77

78 79 80 81

13

$\text{♩} = 138$

82 83 84 85

6/16 4 4 5 9 6

6/16 4 5 7 3

14

$\text{♩} = 126$
5/4 4 4 3 4 3

3/4 4

15

$\text{♩} = 116, \text{♩} = 58$
6/8 4

6/8 4 4 4

6/8 4 4 4

Musical notation for the first system, measures 1-4. The music is written in treble and bass clefs. The melody in the treble clef consists of eighth notes and quarter notes. The bass line in the bass clef consists of quarter notes and eighth notes. The key signature has one flat (B-flat).

16

$\text{♩} = 92$

Musical notation for the second system, measures 5-8. The tempo is marked as quarter note = 92. The music features triplets in both the treble and bass clefs. The treble clef has a key signature change to two flats (B-flat and E-flat) in measure 6. The bass clef continues with the previous key signature.

Musical notation for the third system, measures 9-12. The music continues with triplets in both hands. The treble clef has a key signature change to one flat (B-flat) in measure 10. The bass clef has a key signature change to two flats (B-flat and E-flat) in measure 11.

Musical notation for the fourth system, measures 13-16. The music continues with triplets. The treble clef has a key signature change to two flats (B-flat and E-flat) in measure 13. The bass clef has a key signature change to one flat (B-flat) in measure 14. The system ends with a double bar line.

Musical notation for the fifth system, measures 17-20. The music continues with triplets. The treble clef has a key signature change to one flat (B-flat) in measure 17. The bass clef has a key signature change to two flats (B-flat and E-flat) in measure 18. The system ends with a double bar line.

17

$\text{♩} = 84$

Musical notation for the sixth system, measures 21-24. The tempo is marked as quarter note = 84. The music continues with triplets in both hands. The treble clef has a key signature change to two flats (B-flat and E-flat) in measure 21. The bass clef has a key signature change to one flat (B-flat) in measure 22. The system ends with a double bar line.

First system of musical notation, consisting of two staves. The upper staff features a triplet of eighth notes in the first measure, followed by a series of eighth notes. The lower staff contains a continuous eighth-note accompaniment.

Second system of musical notation, consisting of two staves. The upper staff has a triplet of eighth notes in the first measure, followed by a half note and a quarter note. The lower staff continues the eighth-note accompaniment.

18

Third system of musical notation, consisting of two staves. The upper staff begins with a tempo marking $\text{♩} = 152$ and the instruction *leggiero*. It features a quarter rest followed by a quarter note, then a group of four eighth notes. The lower staff continues the eighth-note accompaniment.

Fourth system of musical notation, consisting of two staves. The upper staff has a group of four eighth notes. The lower staff continues the eighth-note accompaniment.

Fifth system of musical notation, consisting of two staves. The upper staff has a quarter rest followed by a quarter note, then a group of four eighth notes. The lower staff continues the eighth-note accompaniment.

Sixth system of musical notation, consisting of two staves. The upper staff has a quarter rest followed by a quarter note, then a group of four eighth notes. The lower staff continues the eighth-note accompaniment.

66

19

$\text{♩} = 50$

66 67 68

69 70 71

72 73 74

75 76 77

78 79 80

20

$\text{♩} = 176$

21

$\text{♩} = 126$

22

$\text{♩} = 144$

68

23

$\text{♩} = 63$

24

$\text{♩} = 108$

25

$\text{♩} = 92, \text{♩} = 58$

Musical score for measures 24-26. The top staff is in treble clef and the bottom staff is in bass clef. The key signature has one sharp (F#) and the time signature is 3/4. Measures 24 and 25 are in 3/4 time, and measure 26 is in 2/4 time. The music features a complex melodic line with many quintuplets (marked with a '5' and a bracket) and a bass line with triplets (marked with a '3' and a bracket).

26

Musical score for measures 27-29. The top staff is in treble clef and the bottom staff is in bass clef. The key signature has one sharp (F#) and the time signature is 4/4. The tempo is marked as $\text{♩} = 96$. The music features a complex melodic line with many quintuplets (marked with a '5' and a bracket) and a bass line with eighth notes.

Musical score for measures 30-32. The top staff is in treble clef and the bottom staff is in bass clef. The key signature has one sharp (F#) and the time signature is 4/4. The music features a complex melodic line with many quintuplets (marked with a '5' and a bracket) and a bass line with eighth notes.

27

Musical score for measures 33-35. The top staff is in treble clef and the bottom staff is in bass clef. The key signature has one sharp (F#) and the time signature is 4/8. The tempo is marked as $\text{♩} = 80$. The music features a complex melodic line with many quintuplets (marked with a '5' and a bracket) and a bass line with eighth notes.

Musical score for measures 36-38. The top staff is in treble clef and the bottom staff is in bass clef. The key signature has one sharp (F#) and the time signature is 8/8. The music features a complex melodic line with many quintuplets (marked with a '5' and a bracket) and a bass line with eighth notes.

Musical score for measures 39-41. The top staff is in treble clef and the bottom staff is in bass clef. The key signature has one sharp (F#) and the time signature is 4/4. The music features a complex melodic line with many quintuplets (marked with a '5' and a bracket) and a bass line with eighth notes and triplets (marked with a '3' and a bracket).

70

28

♩ = 72

7

29

♩ = 120

7

7

♩ = 88

30

Musical notation for measures 30-31. The system consists of two staves. The top staff is in 4/8 time and features a melodic line with several groups of seven sixteenth notes, each marked with a bracket and the number '7'. The bottom staff is in 4/8 time and provides a bass line with eighth and sixteenth notes. The key signature has one flat (B-flat).

Musical notation for measures 32-35. The system consists of two staves. The top staff has a melodic line with a group of seven sixteenth notes (marked '7') in measure 32, followed by a whole rest in measure 33, and then continues with eighth and sixteenth notes. The bottom staff provides a bass line with eighth and sixteenth notes. The key signature has one flat (B-flat).

31

♩ = 160

Musical notation for measures 36-37. The system consists of two staves. The top staff is in 6/8 time and features a melodic line with a group of seven sixteenth notes (marked '7') in measure 37. The bottom staff is in 6/8 time and provides a bass line with eighth notes. The key signature has one flat (B-flat).

Musical notation for measures 38-41. The system consists of two staves. The top staff is in 9/8 time and features a melodic line with a group of seven sixteenth notes (marked '7') in measure 38, followed by two groups of two eighth notes (marked '2') in measures 39 and 40, and then another group of seven sixteenth notes (marked '7') in measure 41. The bottom staff is in 9/8 time and provides a bass line with eighth and sixteenth notes. The key signature has one flat (B-flat).

Musical notation for measures 42-45. The system consists of two staves. The top staff is in 9/8 time and features a melodic line with a group of seven sixteenth notes (marked '7') in measure 43. The bottom staff is in 9/8 time and provides a bass line with eighth and sixteenth notes. The key signature has one flat (B-flat).

Musical notation for measures 46-49. The system consists of two staves. The top staff is in 9/8 time and features a melodic line with a group of seven sixteenth notes (marked '7') in measure 46. The bottom staff is in 9/8 time and provides a bass line with eighth and sixteenth notes. The key signature has one flat (B-flat).

♩ = 112

32

legato

stacc.

♩ = 138

33

First system of musical notation, consisting of a treble and bass staff. The bass staff features several five-finger patterns (marked '5') and rests.

Second system of musical notation, consisting of a treble and bass staff. The bass staff features several five-finger patterns (marked '5') and rests.

Third system of musical notation, consisting of a treble and bass staff. The bass staff features several five-finger patterns (marked '5') and rests.

34

Fourth system of musical notation, consisting of a treble and bass staff. The treble staff has a tempo marking of quarter note = 132. The system includes triplets (marked '3') and a dynamic marking of *pp*.

Fifth system of musical notation, consisting of a treble and bass staff. The treble staff includes a triplet (marked '3') and a dynamic marking of *pp*.

Musical notation for the first system, measures 74-77. The system consists of two staves. The upper staff has a treble clef and a key signature of two flats. It features a triplet of eighth notes in measure 74, a quarter rest in measure 75, and another triplet of eighth notes in measure 77. The lower staff has a bass clef and a key signature of two flats. It contains eighth-note patterns throughout. A time signature change to 8/8 occurs in measure 76. A bracket above the lower staff in measure 76 is labeled with the mathematical expression $\frac{3 + 2 + 3}{8}$.

Musical notation for the second system, measures 78-81. The system consists of two staves. The upper staff has a treble clef and a key signature of two flats. It features a triplet of eighth notes in measure 79. The lower staff has a bass clef and a key signature of two flats. It contains eighth-note patterns throughout. A time signature change to 7/8 occurs in measure 80.

Musical notation for the third system, measures 82-85. The system consists of two staves. The upper staff has a treble clef and a key signature of two flats. It features a tempo marking $\text{♩} = 112$ above the first measure. The system is in 7/8 time. The upper staff has five-measure rests in measures 82, 83, and 84, with a five-fingered scale starting in measure 85. The lower staff has a bass clef and a key signature of two flats. It contains eighth-note patterns throughout. The number 35 is written to the left of the system.

Musical notation for the fourth system, measures 86-89. The system consists of two staves. The upper staff has a treble clef and a key signature of two flats. It features a five-measure rest in measure 86, followed by a five-fingered scale starting in measure 87. The lower staff has a bass clef and a key signature of two flats. It contains eighth-note patterns throughout. A time signature change to 5/8 occurs in measure 89.

Musical notation for the fifth system, measures 90-93. The system consists of two staves. The upper staff has a treble clef and a key signature of two flats. It features a five-measure rest in measure 90, followed by a five-fingered scale starting in measure 91. The lower staff has a bass clef and a key signature of two flats. It contains eighth-note patterns throughout. A time signature change to 7/8 occurs in measure 92.

Musical notation for the sixth system, measures 94-97. The system consists of two staves. The upper staff has a treble clef and a key signature of two flats. It features a five-measure rest in measure 94, followed by a five-fingered scale starting in measure 95. The lower staff has a bass clef and a key signature of two flats. It contains eighth-note patterns throughout. A time signature change to 8/8 occurs in measure 96. A bracket above the lower staff in measure 96 is labeled with the mathematical expression $\frac{3 + 2 + 3}{8}$.

36 *leggiero* $\text{♩} = 144$

7/8 5/8 7/8

7/8 3/4 3/4

3/4 5/8 7/8

5/8 5/8 5/8

5/8 5/8 5/8

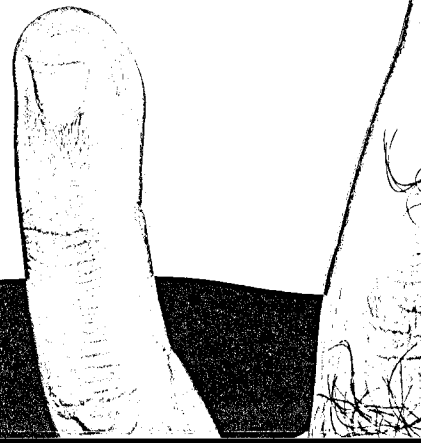


SUMMARY CHART

10			24	25				
9		20		20-21	29-32			
8			20-21		29-30	20-21		
7		25-28	27	25-28	29-32			
6		20	20-21	20-21	31			20
5		24-25, 28	22-24, 28	24-25, 28			29, 32	
4			16-19		29, 31	21	29, 31	
3		9-11		13-16	29-31			
2			11-12		29, 31			
1								
	1	2	3	4	5	6	7	8

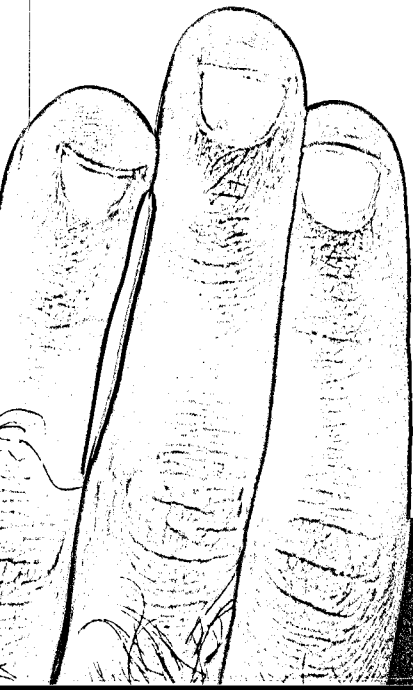
The chart shows the polyrhythmic combinations dealt with in this book. The vertical series of numbers on the far left shows the upper voice component of the polyrhythm, and the horizontal series at the bottom of the diagram shows the lower voice component. The numbers in the co-ordinate system refer to the page in the text where the polyrhythm in question is explained. The references in the prose part then lead to the corresponding exercises.

A given polyrhythm and its inversion will be symmetrically placed, relative to the diagonal line formed by the grey squares between the combination 1 - 1 and 8 - 8. The squares to the left of this central axis correspond with those polyrhythmic situations where the upper voice moves faster than the lower voice, and vice versa for the squares to the right. In practice, the former of these two types is the more common, and so the left side shows the greater number of combinations (21 on the left as opposed to 9 on the right).



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